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MINIMAL MAHLER MEASURE OF AN ALGEBRAIC INTEGER IN A QUADRATIC FIELD

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The Mahler measure of a polynomial with integer coefficients is defined as the product of the absolute value of the leading coefficient and the absolute values of the roots of the polynomial that are outside the unit circle. If α is a nonzero algebraic number, that is, a root of an irreducible polynomial with integer coefficients then the Mahler measure of α is defined to be the Mahler measure of a polynomial. An algebraic integer is defined as a root of an irreducible polynomial with integer coefficients and leading coefficient one. This paper presents the derivation of the smallest possible value of the Mahler Measure $M(\alpha)$ for an algebraic integer α that lies in the quadratic field $Q(\sqrt{m})$ where m is a square free integer.

In 1933, D. H. Lehmer posed his famous question referred to in the literature as Lehmer's conjecture on Mahler measure of a polynomial with integer coefficients. This famous problem, questions the existence of a nonzero algebraic number α such that $M(\alpha) < 1 + \varepsilon$ for every $\varepsilon > 0$, where α is not a root of unity. Also, Lehmer found that the smallest value of Mahler measure $M(\alpha)$ larger than one, associated with the roots of irreducible polynomial $f(x) = 1 + x - x^3 - x^4 - x^5 - x^6 - x^7 + x^9 + x^{10}$ was $M(\alpha) = \alpha_0 = 1.17628...$ where α_0 is the largest real root of the above polynomial.