OPTIMAL PUMP SCHEDULING IN A WATER DISTRIBUTION SYSTEM USING BRANCH-AND-CUT METHOD IN MIXED INTEGER PROGRAMMING

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Introduction

In a water distribution system, one of the main elements of the operational cost is the high consumption of electricity in pumping. A pump station in a water supply system comprises of a set of different pumps. These pumps are used to supply water into the elevated storage in order to satisfy the water demand for the community and this system must satisfy several hydraulic and technical restrictions. Therefore, an optimal pump schedule (OPS) can be expressed as a set of pump combinations that optimizes particular objective (cost of energy), while fulfilling constraints in the water distribution system. Thus, the initial cost of implementing the OPS is at a bare minimum, as it does not change the current infrastructure of the existing system.

Considering the complexity of the system, the computational potential available and the practical aspects are the issues of pump scheduling problem (PSP) that have been subjected to increase research in recent years. Several researchers have suggested different approaches. A very sophisticated study on optimal control for the water pumping system is presented and discussed by Ormsbee *et al.*, is considered and examined with linear and non-linear programming (Ormsbee and Lansey, 1994). The main objective of this study is to establish a mathematical model to optimize the pump scheduling for a water distribution system while satisfying the conditions imposed by the country (*eg.* the electricity tariff in Sri Lanka). The other objective is to develop a computer program to obtain the optimal solution while minimizing the computational time. It can be noticed that the model is a single objective linear model where the decision variables are binary integers. Hence, the proposed optimization method is a hybrid of two Mixed Integer Linear Programming (MILP) algorithms. Finally, the data obtained from an existing water distribution system is fed into the model and the OPS is found after solving it.

Problem definition

The water distribution system consists of one reservoir (storage tank), one potable water source and a set of fixed speed pumps. The optimization schedule is set to one day which is divided into intervals of one hour period. Feasible solution of the OPS problem is coded with binary numbers which gives the state (on/off) of each pump combination during a particular time interval. This combination of pumps will supply water from the sources to the reservoirs, from which the consumer demands are drawn. The proposed objective function, Equation (1), is based on the electrical tariff structure imposed by the Ceylon Electricity Board. The water volume of the storage tank must be operated within a minimum (LV) and a maximum (UV) volumes. The volumes of water in the tanks at the end of each time interval should not be less than the demand of the successive interval in order to achieve periodicity between supply and demand.

Mathematical model formulation

The Mathematical model of the PSP presented by Gomesz *et al.*, is modified and used in this research work (Gomesz and Daundasekera, 2009).

The objective function of the modified model: Minimize the energy cost (1),

$$C_L\sum_{i=1}^{16}\sum_{j=1}^{P}X_{ij}E_j+C_H\sum_{i=17}^{22}\sum_{j=1}^{P}X_{ij}E_j+C_L\sum_{i=23}^{24}\sum_{j=1}^{P}X_{ij}E_j \qquad (1)$$

where i= time interval index, j = pump combination index, X_{ij} = j^{th} pump combination activates in the i^{th} time interval, E_j = consumed electricity of the pump combination j, P= Number of pump combinations, C_L=unit charge of electrical energy in the low cost period, C_H = unit charge of electrical energy in the high cost period.

The following operational constraints are taken into consideration to formulate the model:

The volume in each time interval (V_i) can be expressed as:

$$V_i=\begin{cases}V_0+WP_{ij}-D_i & i=1\\ V_{i-1}+WP_{ij}-D_i & i=2...24\end{cases}, \qquad (2$$

where WP_{ij} = pumped volume of water by the j^{th} pump combination in the i^{th} time interval, D_i = consumers' water demand in the i^{th} time interval, V_0 = initial volume.

The bound on the water volume of the tank in the i^{th} time interval:

$$LV\leq V_i\leq UV. \qquad (3)$$

Since only one pump combination can be activated for a given time interval,

$$\sum_{j=1}^{P}X_{ij}=1;\quad X_{ij}\in\{0,1\};\ i=1,\ldots,24. \qquad (4)$$

It is requested that the total volume deficit should be zero. That is,

$$\sum_{i=1}^{24}\Delta V_i;\quad \Delta V_i=\begin{cases}V_i-IV & i=1\\ V_i-V_{i-1} & i=2,\ldots,24\end{cases} \qquad (5)$$

where, $V_i\leq UV$.

Methodology

The decision variables (X_{ij}) of the Mathematical model of the PSP are binary. The formulated model can be identified as MILP and hence, the Branch-and-Cut algorithm is applied to solve the model. The Branch-and-Cut algorithm is a combination of Branch-and-Bound and Cutting Plane algorithms. It can be shown that the Branch-and-Cut algorithm reduces the computational time to reach the optimal solution, which does not require specialized branching strategies and can be easily implemented. We used the version 4.35 of the GLPK routines (GNU linear programming kit), which implements the Branch-and-Cut algorithm for solving MILP. Three main functions were executed to develop the C-codes to obtain the solution of the formulated model. The first function reads the LP problem and the core function checks the feasible optimal solution with the Simplex algorithm. If the optimal solution is reached, the programme initializes the integer optimization and solves the problem. Finally, the solution is written to a text file. Moreover, the results given by the developed programme (compile in C Language) have been tested with the commercial optimization software LINGO (version 11.0).

The case study

The sample PSP presented by Sotelo *et al.*, (2004) is used to test the applicability of the model and the programme. The system consists of one tank (LV = 2600 m^3, UV= 18200 m^3) and three fixed speed pumps (pumps' capacities are in the Table 1). The model has 192 decision variables ($2^3\times24$).

Table 1. Pumps with capacities

Pump	Volume Pumped (m^3/h)	Electricity Consumed (kWh)
P1	260	828
P2	445	1440
P3	595	1800

The consumer demand profile of a day is assumed from the Sotelo *et al.*, sample system (Figure 1).

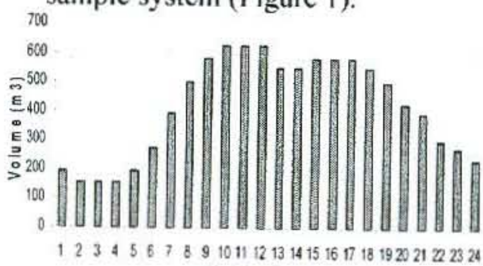

Figure 1. System water demand

Results and Discussion

The initial water volume in the reservoir at the beginning of an optimization period is also the water level that is to be achieved again at the end of the period. As shown in Figure 2, when initial volume of the system increases V_{24} (the end volume) fluctuates between 2600 m^3 and 3000m^3.

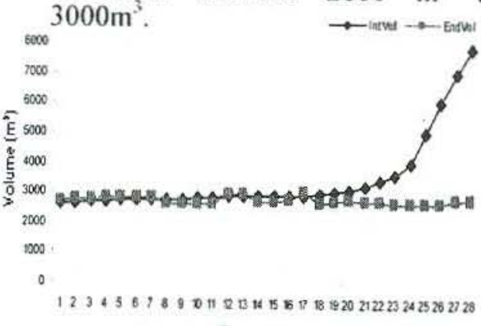

Figure 2. Initial and end volumes

We have determined that the initial volume V_0 =2860 m^3 gives the stable optimal solution for the considered PSP. The Figure 3 shows the pumped water volume in each interval

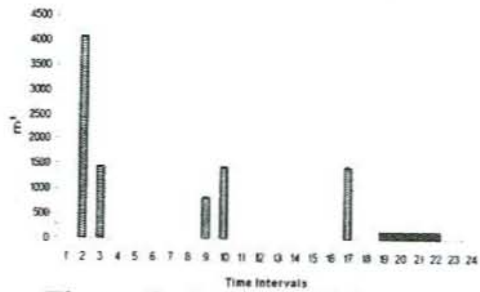

Figure 3. Pumped Volume of the schedule

The cost for the OPS is given by the programme is Rs 28,950. According to the Figure 3, it can be observed that the water pumps are operated during the off-peak tariff periods resulting in cost savings.

Conclusion

In this study, the Branch-and-Cut algorithm is coded to solve the model. The test example shows that the OPS can be easily implemented without changing existing pumps and system configuration. The stable initial volume leads to maintain a continuous distribution system.

References

Gomesz, T.D.A. and Daundasekera, W.B. (2009). Optimizing the pump operations in a potable water distribution system using Branch-and-Bound method in Linear programming. Proceedings of the national symposium on natural resources management, Sabaragamuwa University of Sri Lanka, Belihuloya.

Ormsbee, L.E and Lansey, K. E. (1994). Optimal control of water supply pumping systems. Journal of Water Resource Planning and Management, 120: 237-252.

Sotelo, A., Lucken, C.V. and Benjam, Y.B.(2004). Pump scheduling optimization using asynchronous parallel evolutionary algorithms. Journal of CIEI electronic,7(2): 19-39.