Computation of Bed Shear Stress in Unsteady Open Channel Flows

K.P.P. Pathirana*, P.C. Ranasinghe and U.R. Ratnayake

Department of Civil Engineering, Faculty of Engineering, University of Peradeniya

Introduction

An accurate computation of bed shear stress is important in estimating sediment transport rates in open channels, rivers and streams. The computation of bed shear stress in steady, uniform flow has been extensively investigated by many researchers. Very few studies have been reported in the past investigating the bed shear stress in unsteady flows and most of the studies do not provide conclusive results. In general, the flow in rivers and streams is unsteady and non-uniform. The computation of bed shear stress in this type of flows is still based on the formulations that are developed for steady, uniform flow conditions. The present study is aimed at investigating the estimation of bed shear stress in unsteady open channel flow over rough bed using extensive laboratory experiments (Tu and Graf, 1993).

Methodology

Using the partial differential equations developed by *de Saint Venant* for unsteady flows, shear velocity of unsteady flow is written as (Henderson, 1996);

$$u_{*un} = \sqrt{gy\left[S - \frac{\partial y}{\partial x} - \frac{U}{g}\frac{\partial U}{\partial x} - \frac{1}{g}\frac{\partial U}{\partial t}\right]} \qquad (1)$$

where, U is the depth-averaged velocity, y is the water depth and S is the channel bottom slope. The unsteady flow considered in the proposed study is generated by passing hydrographs in an open channel and hence, the derivatives $\partial y/\partial x$ and $\partial U/\partial x$ in the above equation are changed to time derivatives $\partial y/\partial t$ and $\partial U/\partial t$ using the wave velocity concept and u_* is presented as;

$$u_{*un} = \sqrt{\left[Rg\left(\frac{1}{C}\frac{\partial y}{\partial t} + S\right)\right] - \left[R\frac{\partial U}{\partial t}\left(1 - \frac{U}{C}\right)\right]} \qquad (2)$$

Where, $C = U + \sqrt{gy}$ and R is hydraulic radius.

When passing a hydrograph in the channel, it has been shown that the rate of change of water level is more significant than that of mean velocity and, as a result, the contribution of the second term in Eq.(2) for estimating u_* can be assumed to be very small. Therefore, shear velocity in unsteady flows can be expressed by the following simplified equation,

$$u_{*un} = \sqrt{\left[gRS + \frac{1}{C}gR\frac{\partial y}{\partial t}\right]} \qquad (3)$$

Experimental set-up and procedure

The experiments were carried out in a 10 m long, 0.4 m wide, 0.5 m deep rectangular, re-circulating, tilting flume. The size of natural river sand used for the experiments is between 2 mm and 8 mm. The fixed bed was prepared by gluing a single layer of sand on the channel bed with cement slurry. The base flow of a known quantity was first allowed to flow over the fixed bed. Using an additional pump in addition to those used in the re-circulating system and with preset valve opening and timing the pump running durations, six hydrographs were passed over the steady base flow. Each hydrograph was repeated several times to ensure repeatability. The experiments were repeated for four different channel slopes and four different base flow discharges, with six hydrographs each, leading to a total of 96 test runs. The water depths were measured continuously during the tests using pressure sensors and all observations were logged into a computer using an Analogue-Digital converter.

Results and discussion

Unsteadiness of flows was quantified using a hydrograph parameter (*HYDP*), which is defined as:

$$HYDP = \frac{2(y_p - y_b)y_p}{(u_{b*}DT)^2} \qquad (4)$$

Where, y_b is the water depth in the base flow, y_p is the water depth at peak, u_{*b} is the shear velocity in the base flow and DT is the duration of hydrograph. The *HYDP* of test cases varied between 3.17×10^{-4} (most unsteady) and 1.52×10^{-5} (least unsteady). As a typical result of the unsteady flow runs, the time variation of water depth and shear velocities during the passage of a hydrograph is shown in Figure 1. Figure 2 compares the u_* values based on the

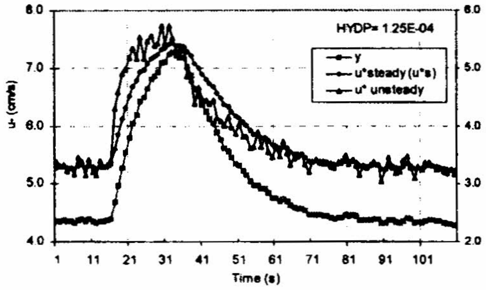

Figure 1. Variation of water depth and shear velocity in an unsteady flow run

steady and unsteady formulae during the passage of a hydrograph with a *HYDP* of 3.17×10^{-4}. The difference in shear velocities derived from the steady and unsteady flow equations seems to increase significantly for the rising and falling limbs of the hydrograph as illustrated in Figure 2. It clearly shows that the shear velocities in the rising limb are higher than that in the falling limb.

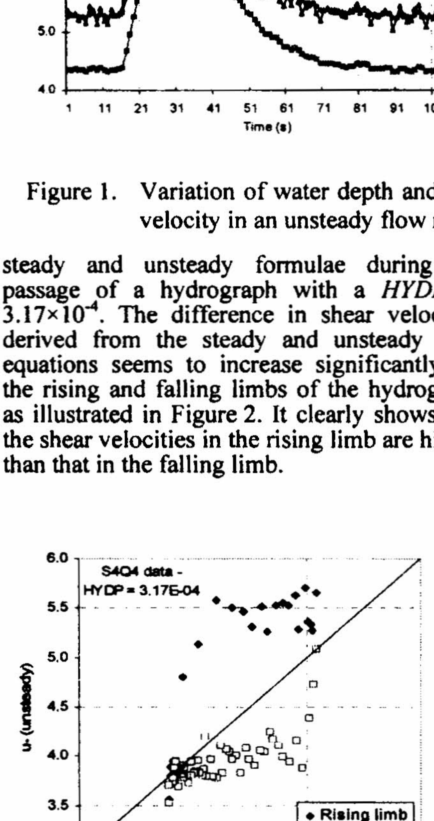

Figure 2. Comparison of shear velocities computed by steady and unsteady state formula

It can be seen from Figure 3 that the maximum difference in u_* computed by the two methods is generally larger in the rising limb than that in the falling limb and that it always remains less

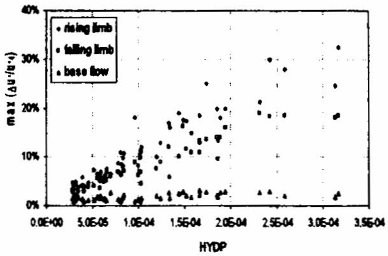

Figure 3. Variation of maximum $(\Delta u_*/u_{*s})$ with *HYDP* in the rising and falling branches of the hydrographs

than 3% in the base flow. The maximum value of $(\Delta u_*/u_{*s})$ increases with *HYDP* indicating that the error can be as high as nearly as 30% and 20% in the rising and falling limbs, respectively when *HYDP* is high. The difference in shear velocities computed by steady and unsteady formulae was derived as,

$$\frac{\Delta u_*}{u_{*s}} = \left(\left|\frac{u_{*un}-u_{*s}}{u_{*s}}\right|\right) = \left|\sqrt{1+\frac{1}{US}\left[\frac{Fr}{1+Fr}\right]\frac{\partial y}{\partial t}}-1\right| \quad (5)$$

Where, *Fr* is the Froude number and u_{*s} is the shear velocity in steady uniform flow, which is given by $u_{*sl} = (\tau_0/\rho)^{0.5}$.

Conclusions

The behaviour of bed shear stress in unsteady flow was studied using extensive laboratory experiments over a rough bed. The shear velocity in unsteady flow derived from *de Saint Venant* equations was compared with the steady state formula. The shear velocity is usually larger in accelerating flow than that in decelerating flow. The difference in shear velocities computed by the steady and unsteady flow equations was quantified and compared with the experimental results. A relationship between steady and unsteady shear velocities has been derived in terms of hydraulic parameters of the problem, which can be used to quantify the error in using steady state formula of u* for unsteady flows.

References

Henderson, F.M. (1996) Flood waves in prismatic channels, *Journal of Hydraulic Engineering*, ASCE, 89(4), 39–67.

Tu, H. and Graf, W.H. (1993) Friction in unsteady open-channel flow over gravel beds, *Journal of Hydraulic Research*, 31(1), 99–110.