IMPROVED ESTIMATION OF PARAMETERS IN PROBABILITY DISTRIBUTIONS WITH KNOWN PRIOR INFORMATION

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ABSTRACT

In this thesis a study of the theoretical and practical aspects of the optimal shrunken estimation for unknown parameters with known prior information, introduced by Searls in 1964 is undertaken. And consequently several authors Khan (1968), Gleser and Healy (1976), Arnholt and Hebert (1995) and recently Wencheko and Wijekoon (2005) have shown that a uniformly better (in the mean square error sense) estimator than the usual unbiased estimator can be constructed by considering the mean square error (MSE) of a constant multiple of the unbiased estimator when coefficient of variation is known.

This work mainly concerns with the results presented by Gleser and Healy (1976) as well as Arnholt and Hebert (2001), whose results have been generalized in this study to the problem of estimating an unknown parametric function or vector, through certain criterion. The classical problem of estimating an unknown parameter is generally solved by using the minimum mean squared error (MMSE) estimator, and this research also adopts this approach, and demonstrates analytically since the found techniques perform the optimum method in terms of mean square error (MSE). Then the methods for finding these estimators are discussed and it is demonstrated that the proposed estimators outperform the optimum approach in many cases.

A shrinkage framework to perform optimal estimators of parameter estimation is presented in the literature. The formulation allows the incorporation of existing prior information on the probabilistic models in a consistent way. The method is based on the classical approach that allows a straightforward utilization of the prior information. This resulted in an extension of the shrinkage technique which forms the main body of this work. This novel technique is referred as *optimal shrinkage estimation* (OSE).

It is shown that mean square error optimal shrinkage estimation (MSE-OSE) method is a simple, flexible, and attractive OSE-method. The MSE-OSE method is simple because it is based on the well-known principle of parameter estimation. In this MSE-OSE method, the estimation of the parameters is based on sufficient statistic of parametric (probability) models. The method is flexible as it works with a user-defined known constant (k). Different scalar classes can be considered but they all automatically lead to a unique OSE of parameters. Using this method, the uniformly minimum mean square error (UMMSE) estimator $k\left(k^2+\tau^2\right)^{-1}\phi(T(X))$ of parameter $g(\theta)$ can be obtained by choosing the complete sufficient statistic $T(X)$ as the estimator class of the parameter θ, when the ratio known as "*WIJLA*" ratio $\tau^2=[g(\theta)]^{-2}Var[\phi(T(X))]$, is independent of θ. Further, the result of combining estimators for $g(\theta)$ is presented when θ has more than one sufficient statistic.

Using these results the OSE in a certain class for both population mean and variance of some probability distributions can be obtained when additional information such as coefficient of variation, kurtosis or skewness is known. It is proposed that a general OSE for natural exponential families with quadratic variance functions covers the binomial, Poisson, negative binomial, gamma, and generalized hyperbolic secant distributions. The mean square errors of these estimators were compared, and a numerical illustration has been done using the scaled mean square error loss as used by Kanfuji and Iwase (1998) to understand the efficiency of the estimators with increasing sample size. Finally, an estimation of parameter vector as an extension of the estimators in the univariate parametric model is derived by applying the ideas mentioned above to estimators in the multivariate parametric model. Therefore, by using this result, the OSE for mean vector of some probability models is found when multivariate coefficient of variation is available.