Determining the Optimal Order Quantity Using a Simulation Model

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Introduction

Simulation models are logical descriptions of the interrelationships among elements of a decision problem or the sequence of events that occur in a system over time. The major advantage of a simulation model is its ability to incorporate uncertainty in one or more input quantities. It captures probabilistic behavior and allows one to estimate probability distributions and key statistics such as mean, variance, etc. that cannot be derived analytically.

Due to the fluctuation of the sales according to the market demand, effectiveness of the company promotional activities, seasonal variations and competition with other suppliers, the quantity of sales in any given period is not fixed. Therefore, the company has to make an order under certain uncertainties.

This paper uses a simulation model to determine the optimal order quantity for a wholesale supplier for a certain period of time by analyzing the quantified risk involved in each demand quantity decision.

Methodology

The past sales data of the item, non-woven interlining, were analyzed using the method of linear regression analysis to find an approximate quarterly sales distribution for year 2007. Furthermore, with the help of @Risk simulation package (Albright *et al.*, 2002), the risk involved in each demand quantity was analyzed to predict the optimal stock for each quarter in 2007 which satisfies the company requirements.

Data collection and preprocessing

Monthly sales data of the non-woven interlining for the past 5 years were used. Since our objective is to find the optimal order quantities for each quarter of the year 2007, the quarterly sales in the past 5 years were calculated using the collected data. Then, by categorizing these data according to the quarters, the respective quarterly sales for 2007 were obtained.

Results

Regression analysis

The expected quarterly demands for each quarter in 2007 given in Table 1 were calculated with the use of the summary output obtained from the linear regression analysis method (Utts and Heckard, 2004). Next, the error in this quantity was taken to be normally distributed with mean (μ) = 0 and standard deviation (σ) = standard error obtained in the summary output given in Table 2. As shown in Table 3, the actual demand for each quarter in 2007 was taken to be the (expected demand) + (error), where error ~ $N(0,\sigma)$.

Table 1. The expected quarterly demands

Quarter	Exp demand
Quarter 1	336931.4 m
Quarter 2	296937.2 m
Quarter 3	294826.7 m
Quarter 4	459949.5 m

Table 2. Error for each quarterly demand

	μ	σ
Quarter 1	0	24029.04 m
Quarter 2	0	15612.01 m
Quarter 3	0	29491.02 m
Quarter 4	0	15196.34 m

Table 3. The actual demand for 2007

Quarter	Predicted actual demand(m)
Quarter 1	336931.4 + Error
Quarter 2	296937.2 + Error
Quarter 3	294826.7 + Error
Quarter 4	459949.5 + Error

Simulation model

With respect to the possible range for error, the actual demand will also fall into a range. Therefore, as shown in Table 4(i), Table 4(ii), Table 4(iii), Table 4(iv) and Table 4(v), a simulation model was built using the Excel spreadsheet expressing the company sales process logically (Evans, 2000). Next, by assigning different values for order quantity and by simulating the demand values given in Table 3, the respective profit distributions were found.

Table 4(i). Cost data

Unit cost(c)	Rs 20.5
Unit price(p)	Rs. 23.575

Table 4(ii). 2007 demand distribution (assumed to be Normal with the following parameters)

	μ	σ
First quarter	336926.6 m	24050.88 m
Second quarter	296930.2 m	15626.32 m
Third quarter	294827.6 m	29435.96 m
Fourth quarter	459950.5 m	15188.3 m

Table 4(iii). Possible order quantities (x_n) (in thousand meters)

	(Decision variable)								
First quarter	250	270	290	310	330	340	350	370	390
Second quarter	230	250	270	290	300	310	330	350	370
Third quarter	190	210	230	250	270	290	300	310	330
Fourth quarter	400	420	440	450	460	480	500	470	480

Table 4(iv). Decision variable (x_n)

	order quantity (m)
First quarter	250000
Second quarter	230000
Third quarter	190000
Fourth quarter	400000

Loan interest rate (r)

0.18

Depreciation rate(d)

2%

Cash balance/Budget(A)

Rs 1000000

Table 4(v). Simulated quantities

	Demand (m)	Revenue (Rs)	Cost (Rs)	Bank loan (Rs)	Interest for the loan(Rs)	Profit (Rs)
First quarter	335626.1	5893750	5125000	4125000	185625	583125
Second quarter	296776.3	5422250	4715000	3715000	167175	540075
Third quarter	297992.9	4479250	3895000	2895000	130275	453975
Fourth quarter	458860.3	9430000	8200000	7200000	324000	906000
					Net profit	2483175

Mathematical model of the simulation model for the n^{th} quarter :

Let
Unit cost = c , Unit price = p ,
Demand = D_n , Order quantity = x_n ,
Revenue = R_n , Cost = C_n , Profit = P_n
Budget = A , Amount of loan = L_n ,
Annual loan interest rate = r ,
Interest for the loan = I_n ,
Depreciation rate of the remaining quantity = d

Decision variable : x_n
Input variables : D_n , c , p

Objective : Max $P_n = R_n - C_n - (I_n + L_n)$
subjected to a minimum risk ,
where
$R_n = c * \min(D_n, x_n)$, $C_n = c * x_n$,
$L_n = C_n - A$, $I_n = (L_n * r) \div 4$

For the year 2007
Unit cost (c) = Current unit cost + cost increment
Unit price (p) = Unit cost * (1 +expected Profit percentage)
Current unit cost = Rs. 20,
d = 2% , r = 18%
Increment in cost ~Triang (0.30 , 0.50 , 0.80)
Expected profit percentage ~ Uniform (14.5%, 15.5%)

By analyzing the summary statistics report, sensitivity analysis report and the tornado charts for the profit distributions, the risk involved in each order quantity was quantified and the order quantities given in Table 5 were taken as the optimal order quantities which gave a relatively high profit for each quarter in 2007.

Discussion

As shown in Table 5, even with the best values for mean profit, there is a risk of getting negative profit. But these preferred outputs have a relatively a few number of influential inputs compared to the other simulations for each quarter. Also, these order quantities give a relatively high mean and a small standard deviation for the profit. Hence, the values 310,000 m, 270,000 m, 250,000 m and 440,000 m can be considered as the optimal order quantities for the non-woven interlining in the 1st, 2nd, 3rd and 4th quarters for the year 2007, respectively.

References

Albright, Winston, Zappe (2002) *Data analysis and decision making with Microsoft EXCEL*, Duxdury Press, 891-977.

Evans, J.R. (2000) *Spreadsheets as a Tool for Teaching Simulation*, Volume 1, University of Cincinnati, Cincinnati,

Utts, J.M. and Heckard, R.S. (2004) *Minds on Statistics*, Second edition, Thomson, 492-512.

Table 5. The order quantities with the maximum mean for the profit

Quarter	Simulation No.	Order Quantity (m)	Minimum of the profit (Rs)	Mean of the profit (Rs)	Maximum of the profit(Rs)
1	4	310,000	-568242	674806.7	755428.2
2	3	270,000	-77841.7	619493.9	663760.1
3	4	250,000	-541476	560140.5	613019.3
4	3	440,000	161871.1	975908.3	1053350