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BOUNDARY INTERPOLATION ON THE UNIT DISK WITH FINITE BLASCHKE PRODUCTS

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Let $\mathrm{D}=\{z\in\mathbb{C}:|z|<1\}$. A finite Blaschke product of degree n is a function of the form $B(z)=\lambda\prod_{k=1}^{n}\frac{z-a_k}{1-\overline{a_k}\,z}$, where $|\lambda|=1$ and $a_k\in\mathrm{D}$ for $k=1,2,\ldots,n$. In 1965, Cantor and Phelps proved the following interpolation theorem:

Theorem : If $\alpha_1,\alpha_2,\ldots,\alpha_n$ be n distinct points on $\partial\mathrm{D}=\{z\in\mathbb{C}:|z|=1\}$ and if $\beta_1,\beta_2,\ldots,\beta_n$ are points on $\partial\mathrm{D}$, then there exists a finite Blaschke product B such that $B(\alpha_k)=\beta_k$ for $k=1,2,\ldots,n$.

Their proof was based on a general result on *n*-transitive family of maps from a semi-group S into itself. In 1980, Younis gave another proof of this theorem, but the degree of the interpolating finite Blaschke product was not the best possible. In 1987, Jones and Ruscheweyh gave a non-constructive proof of the above theorem by considering a similar problem in the half-plane. The degree of the finite Blaschke product there was $n-1$ which, in general is the best possible. (This proof is also sketched in *approximation by Interpolating Blaschke products* by Hjelle.) In 2008, Gorkin and Rhoades proved the above theorem by constructing an interpolating finite Blaschke product of degree $n-1$. In the same year, Perera and Ranasinghe gave a simpler direct proof of this result, but the degree was not the best possible.

It is known that a finite Blaschke product B of degree n with $B(0)=0$ has a representation of theform $\frac{1}{1-B(z)}=\sum_{k=1}^{n}\lambda_k\frac{1}{1-\overline{x_k}z}$, where $\lambda_k>0$, $|x_k|=1$ for $k=1,2,\ldots,n$ and $\sum_{k=1}^{n}\lambda_k=1$. Our aim here is to construct an elegant reduction procedure based on this result which allows us to give a new proof of the above theorem with an interpolating finite Blaschke product of degree $n-1$.