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SECOND ORDER NECESSARY CONDITIONS FOR AN OPTIMIZATION PROBLEM WITH *n*-SET FUNCTIONS

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The concept of optimizing set functions arises in various areas in Mathematics, Engineering and Statistics. The optimization of set functions becomes difficult when the feasible domain is neither open nor convex, and nowhere dense. In 1979, Robert J. T. Morris overcame these difficulties by properly defining notions of convexity and differentiability of a set function. He considered the optimization problem in a measure space (X, A, μ).

In this paper, we consider the following non-linear programming problem involving *n*-set functions:

$$\text{Minimize } F(R_1,\ldots,R_n)$$

Subject to

$$G_i(R_1,\ldots,R_n) \le 0,\ i = 1,\ldots,m,$$
$$(R_1,\ldots,R_n) \in A^n,$$

where A^n is the *n*-fold product of a σ - algebra A of subsets of a given set X, and F and G_i, $i = 1,\ldots, m$, are real-valued functions defined on A^n.

The ideas and results initiated by Morris have been utilized and extended later by others. In particular, the theory developed by Morris has been extended to *n*–set functions.

We define the first order and second order derivatives, and the second order differentiability, of an *n*–set function. The second order differentiability is illustrated by giving an example. We show the existence of the Morris sequence when *n*-set functions are involved. Finally, we define the local optimal and obtain second order necessary optimality conditions for the non-linear programming problem, involving *n*-set functions, mentioned above.