

DIRECT METHODS AND THE PARTICLE METHOD FOR THE GARDNER EQUATION

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Gardner equation, which takes the form, $u_t + \alpha uu_x + \alpha_1 u^2 u_x + \beta u_{xxx} = 0$ is a standard mathematical model for wave propagation, when a strong nonlinearity is experienced. Here $u(x, t)$ describes the wave at place x at time t and the coefficients, α , α_1 and β are determined by the environment density and shear flow stratification. Unlike other nonlinear evolution equations, this particular equation cannot be integrated by classical approach, method of Inverse Scattering Transform. We numerically integrate the equation using spectral methods and exponential Runge Kutta time stepping of order four in MATLAB. The results are compared with the solutions obtained from various integrating techniques such as *cosh* ansatz, *tanh* and *sech* methods. Moreover we discuss the types of the solutions for different parameter values of the equation. For instance, thick solitary wave with flat crest exists only if α_1 is negative, that is, the cubic nonlinear term has opposite polarity to the linear dispersive term.

More importantly we derive the particle method that can approximate the solution the Gardner Equation. Particle methods have become one of the most effective and widely used tools in approximating solutions for partial differential equations. This method considers the solution to be a collection of particles located at some points and carrying masses. The dynamics of the location of the particles and their weights can be described from the equation of evaluation of time. The accuracy of this method will depend on the choice of so called cutoff function and on its width. We have successfully implemented the particle method for the Gardner equation and discuss the validity of this method by considering different values for the coefficients. The numerical integration and the particle method work well for any parameter value. We suggest the particle method as a powerful method to solve Gardner equation.