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ON PRODUCTS OF HALF-PLANE MAPPINGS

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Let $\mathbb{D} = \{z \in \mathbb{C}: |z| < 1\}$ and let f and g be functions analytic in $\mathbb{D}$. Then f is said to be subordinate to g if $f(z) = g(\varphi(z))$ for $z \in \mathbb{D}$, where $\varphi: \mathbb{D} \longrightarrow \mathbb{D}$ is analytic in $\mathbb{D}$ with $\varphi(0) = 0$. This is denoted by $f \prec g$. A trivial modification of the Herglotz representation formula for functions subordinate to half-plane mappings implies that if

$$f \prec \frac{1+cz}{1-z}, \text{ where } |c| = 1, \text{then } f(z) = \int_{\partial\mathbb{D}} \frac{1+cxz}{1-xz}\, d\mu,$$

where μ is a probability measure on the unit circle $\partial\mathbb{D}$.

In 1989, Koepf considered the class of functions p normalized by $p(0) = 1$ and $p \prec \frac{1+cz}{1-z}$ for some $|c| = 1$ and proved that each function of the form

$$p_n(z) = \sum_{k=1}^{n} \lambda_k \frac{1+cx_k z}{1-x_k z}, \text{ where } |x_k| = |c| = 1,\ \lambda_k > 0 \text{ for } k = 1,2,\dots,n \text{ and } \sum_{k=1}^{n} \lambda_k = 1 \quad (*)$$

has a representation of the form

$$p_n(z) = \prod_{k=1}^{n} \left(\frac{1-y_k z}{1-x_k z}\right), \text{ where } |x_k| = |y_k| = 1 \text{ for } k =,2,\dots,n$$

and $\arg x_1 < \arg y_1 < \arg x_2 < \arg y_2 < \cdots < \arg x_n < \arg y_n < \arg x_1 + 2\pi$. $(**)$

In this study we first give a new proof of the above product representation using the following known representation for finite Blaschke products:

If B is a finite Blaschke product with $B(0) = 0$, then $\frac{1+B(z)}{1-B(z)} = \sum_{k=1}^{n} \lambda_k \frac{1+x_k z}{1-x_k z}$,

where $|x_k| = 1$, $\lambda_k > 0$ for $k = 1,2,\dots,n$ and $\sum_{k=1}^{n} \lambda_k = 1$.

We then considered the question of whether each function of the form $(**)$ has a representation of the form $(*)$. We were able to prove it for $n = 2$ directly. Since the computation becomes tedious for $n = 3$ with the direct method, we employed the Herglotz representation formula to prove it. Based on the above results and verification for some more cases using *Mathematica*, we conjecture that each function of the form

$$p_n(z) = \prod_{k=1}^{n} \left(\frac{1-y_k z}{1-x_k z}\right), \text{ where } |x_k| = |y_k| = 1 \text{ for } k = 1,2,\dots,n$$

and $\arg x_1 < \arg y_1 < \arg x_2 < \arg y_2 < \cdots < \arg x_n < \arg y_n < \arg x_1 + 2\pi$
has a representation of the form $p_n(z) = \sum_{k=1}^{n} \lambda_k \frac{1+cx_k z}{1-x_k z}$, where $|x_k| = |c| = 1$, $\lambda_k > 0$ for $k = 1,2,\dots,n$ and $\sum_{k=1}^{n} \lambda_k = 1$.