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DEVELOPMENT AND VALIDATION OF A MATHEMATICAL MODEL FOR ARTIFICIAL BRAIDED PNEUMATIC MUSCLE

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Introduction

Artificial Braided Pneumatic Muscles (ABPM) are widely used in different industrial disciplines, especially in robot-arm manufacturing because of the lesser weight, better sensitivity for reactive forces, similarity to the human muscle operation and the simplicity. The ABPM is primarily a cylindrical air-actuator having a rubber bladder acting as an air seal, surrounded by inelastic fibre mesh to control its expansion. When the ABPM pressurises, the diameter of the cylinder increases whilst the length decreases. (Figure 1)

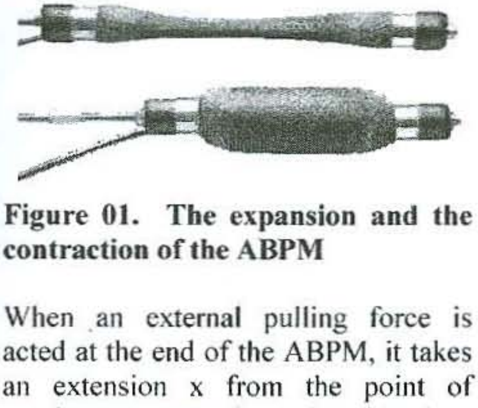

Figure 01. The expansion and the contraction of the ABPM

When an external pulling force is acted at the end of the ABPM, it takes an extension x from the point of maximum contraction. The objectives of the research are to construct a mathematical model that computes the force exerted for a given extension at a known pressure based on reasonable assumptions and to validate the model using actual data (presented by the manufacturers).

Since the innovation of the 'Expandable Cover' in early 1940s by C. R. Johnson and R. C. Pierce, there were renovations of the same concept for different purposes such as using it as an actuator by Gaylord and medical-physical applications by McKibben [Chou et al 1996]. In addition, the development of Mathematical Models such as Gaylord in 1965, Chou et al in 1996, Colbrunn et al in 2001, etc describing the operation have taken place. However, the number of research carried out so far for analytical modelling is insufficient while the available models are complicated and subjected to errors. For example the formula derived by Chou and Hannaford in 1996 contains the braid angle θ in the final model, leading practical difficulties in applying it, as θ varies with extension x and due to the difficulties of taking instantaneous measures. The energy method used by Klute is complicated and the results are not accurate enough compared with the complexity. Colbrunn's results are not accurate as Klute's Model, or the Model described in this paper. Presently Festo Corporation, Bridgestone and Shadow Robot companies manufacture the ABPMs.

Materials and Methods

A prototype ABPM was used to understand the configuration. Because, the length in the ending conical portions is negligible compared to the total length, the pressurised ABPM was assumed as cylindrical. The stress on the rubber bladder was neglected, as it is a highly elastic material having a negligible thickness. First, the hoop and the longitudinal stresses were calculated. Then the tensions were deduced.

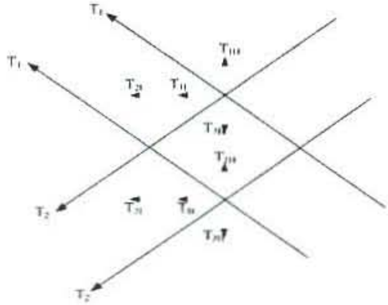

Figure 02. Tensions at a crossing

Let T be the tension of a fibre band caused by an extension x. Let T_1, T_2 indicate the tensions acting along two fibres at a crossing (Figure 2). (T_1 and T_2 are equal in magnitude) Dotted arrows show the resolved components of T_1 and T_2 along the hoop and the longitudinal directions respectively. The summation of the longitudinal components T_{1L}, T_{2L} leads to the longitudinal tension (T_L) that drives the axial motion of the ABPM. The Hoop components T_{1H} and T_{2H} are not cancelling off as they are acting on two fibre bands. In fact, they lead to increase 2θ , the angle between two fibre bands in a crossing, causing the radial expansion. As the total longitudinal tension NT_L (N is the number of fibre bands), drives the axial motion of the ABPM, each T_L leads to decrease 2θ . Therefore, the arithmetic difference between NT_L: $\left(|T_{1L}| = |T_{2L}| = |T_l|\right)$ and the total hoop tensions NT_H: $(|T_{1H}| = |T_{2H}| = |T_h|)$ determines the ABPM operation. Let T_r be the difference of T_l and T_h. Then the force F_x exerted by the ABPM is;

$$F_x = \sum_{r=1}^{N} T_r \cos\theta = N(T_l - T_h)\cos\theta \ (1)$$

Results

Let x – be the extension from the point of maximum contraction.

$$x = (1-k)L - L_m \qquad (2)$$

where L_m is the length of the cylinder at maximum contraction, and L is the length before pressurizing. k is the percentage contraction and the practical observation of 20% was used in simulation as the upper bound of k .

$$\theta = \tan^{-1}\left(nD_x / (L_m + x)\right) \qquad (3)$$

where n – is the number of fibre turns per cylinder length. (n = 2 for the prototype ABPM) D_x is the diameter at an extension, x. Considering the volume consistency at maximum contraction, and at k% of contraction,

$$D_x = D_m \sqrt{L_m / (L_m + x)} \qquad (4)$$

D_m :the diameter at L_m.

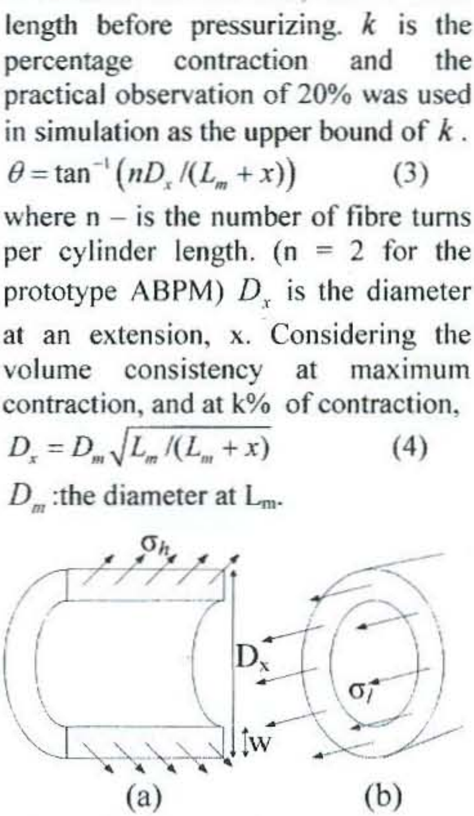

Figure 03. σ_h **and** σ_l

The hoop stress σ_h (Figure 3-a) and the longitudinal stress σ_l (Figure 3-b) are given by the equations (5) and (6).

$$\sigma_h = (D_x - 2w)P / 2w \qquad (5)$$

$$\sigma_l = (D_x - 2w)^2 P / [4w(D_x - w)] \ (6)$$

where P is the actuator pressure, and w is the cross sectional height of a fibre band. The cross section of a fibre band was assumed as rectangular having a width of $2w$ as opposed to the adjacent circular cross sections. The equation (7) holds for any stress. (8), (9) compute the hoop force F_h and the longitudinal force F_l.

$$\sigma = F / A \quad (7)$$

where, F is the force, and A is the cross sectional area.

$$F_h = nNT_h Sin\theta_x \quad (8)$$

$$F_l = nNT_l Cos\theta_x \quad (9)$$

Solving the equations (5), (6), (7), (8) and (9) yields,

$$T_h = \frac{P(D_x - 2w)(L_m + x)}{2nNSin\theta_x} \quad (10)$$

$$T_l = \frac{P(D_x - 2w)^2 (L_m + x) Cos\theta_x}{4nN(D_x - w) Sin^2\theta_x} \quad (11)$$

At maximum contraction (Fig 02) $x = 0$, $T_h = T_L$, $D_x \to D_m$ and $\theta_x \to \theta_m$

$$\Rightarrow T_{kH} = T_{kL} : for, k = 1,2 \quad (12)$$

$$T_{kH} = T_k \sin\theta_m \quad (13)$$

$$T_{kL} = T_k \cos\theta_m \quad (14)$$

By simplifying (12), (13) and (14):

$$\theta_m = 45^0 \quad (15)$$

Discussion

The equations above calculate F_x (in Equation 01). The experimental data variations were taken from the product specifications provided by Bridgestone Co. Ltd. for comparison purposes.

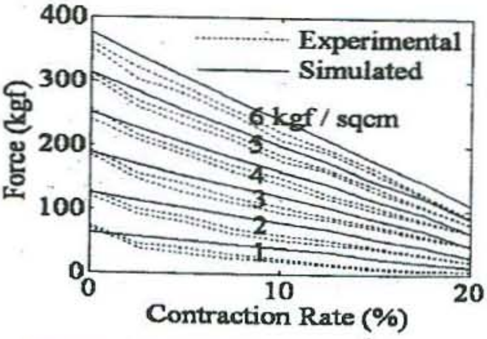

Figure 04. Simulation

The comparison infers that the formula predicts F_x with reasonable accuracy (simulation for 40 cm actuator). The variables w, n and N were assigned in comparison with the prototype of length 35 cm. The model might perform better with precise parameter values. The experimental data show a hysterisis due to the friction

Conclusion

Tensions at the conical ends ignored and it may cause an error. However the longer the cylinder, the lower the ratio of conical length to cylindrical length and therefore the model suits better for longer ABPMs and it has to modify for shorter ABPMs. As the model is sensitive for the accuracy of the parameters, they have to be measured carefully. The hysterisis may be obtained by incorporating the friction.

References:

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