# AN APPLICATION OF THE REPLACEMENT PROBLEM

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## Introduction

The study of replacement is concerned with situations that arise when some item such as machine, men, equipments need replacement due to their deterioration, failure or break down. Physical impairments or malfunctioning of various parts would lead to a decline in the value of the service rendered by the equipment, increase operating and maintenance cost of the equipment. Due to improvements in the existing tools and machinery, obsolescence takes place. Therefore, it becomes uneconomical to continue production with the same equipment under any of the above situations. Hence, the equipments are to be periodically replaced.

The main objective of this research was to find the optimal replacement policy to replace normal Juki machines of an apparel company according to the data gathered from the company. Two policies can be found for the replacement of those machines (Paneerselvam, 2008). They are:

1. Replacement of machines which deteriorate with time.
2. Replacement of machines that do not deteriorate with time, but fail suddenly.

In replacement of machines that deteriorate with time, we must check whether the value of money remains constant or value of money changes with time. If value of money remains constant, then again we must check whether the time is a continuous variable or discrete variable. If value of money changes with time, again we must check whether the resale price is negligible or not (Bernard and Robert, 1995). In this research the optimal replacement policy was found for machines that deteriorates with time where value of money changes with time and the resale price is not negligible.

## Methodology

Annual maintenance costs and resale prices of a machine from year 2000 to 2008 are collected for a machine that deteriorates with time. According to the rate of inflations, the present worth factor is calculated. Then the weighted average costs are calculated and the minimum weighted average cost is preferred. Calculations have been done to determine the optimum replacement period which are given in the Table1.

In Table 1,

$C_{k-}$ : Maintenance cost of machine in year $k$-1.

$v^{k-1}$: Present worth factor in year

$S_k$ : Resale price of the machine in year $k$.

$C_{k-1}v^k$ : Present value of the maintenance cost of the machine in year $k$-1.

$S_kv^k$ : Present value of resale price in year $k$.

$\sum C_{k-1}v^{k-1}$: Cumulated sum of discounted maintenance cost in year $k$-1.

$C+\sum C_{k-1}v^{k-1} - S_k v^k$ : Total present worth of the costs, where $C$ is the initial cost.

$\sum v^{k-1}$ : Cumulative sum of present worth factor.

$W_k$ : Weighted average cost in year $k$.

The minimum weighted average cost occurred in year 2008, but any conclusion cannot be made since there is a possibility of decreasing the weighted average cost after year 2008. So, the trend line must be drawn and the optimality must be checked. These are shown in Figure 1.

According to the plot of weighted average cost vs. year, it shows that the minimum weighted average cost occurs at 9.64982.

**Results**

According to the calculation of weighted average costs, it is observed that the weighted average costs are decreasing with time and least weighted average cost occurs at year 2008 (the 9th year).Then the graph (plot of weighted average cost vs. year) shows that the minimum weighted average cost occurs at 9th year (Figure 2). Therefore, the machine should be replaced after every 9th year.

**Table 1. Optimum replacement period**

| Year | Main. cost $C_{k-1}$ | p.w.f $v^{k-1}$ | $v^k$ | $C_{k-1}v^{k-1}$ | ∑Ck-1vk-1 | Resale price $S_k$ | $S_k v^k$ | Total present worth $160{,}000+\sum C_{k-1}v^{k-1}-S_k v^k$ | $\sum v^{k-1}$ | Weighted average cost $W_k$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 2000 | 18,000 | 1.000 | 0.894 | 18,000 | 18,000 | 150,400 | 134,457.6 | 43,542.4 | 1.000 | 43.542 |
| 2001 | 18,375 | 0.894 | 0.799 | 16,427.3 | 34,427.3 | 141,376 | 112,959.4 | 81,467.8 | 1.894 | 43.014 |
| 2002 | 18,570 | 0.799 | 0.714 | 14,837.4 | 49,264.7 | 132,893 | 94,885.6 | 114,379.1 | 2.693 | 42.473 |
| 2003 | 18,650 | 0.714 | 0.638 | 13,316.1 | 62,580.8 | 124,920 | 79,699.0 | 142,881.8 | 3.407 | 41.938 |
| 2004 | 19,200 | 0.638 | 0.570 | 12,249.6 | 74,830.4 | 117,424 | 66,931.7 | 167,898.7 | 4.045 | 41.508 |
| 2005 | 19,750 | 0.570 | 0.509 | 11,257.5 | 86,087.9 | 110,378 | 56,182.4 | 189,905.5 | 4.615 | 41.150 |
| 2006 | 20,575 | 0.509 | 0.455 | 10,472.7 | 96,560.6 | 103,755 | 47,208.5 | 209,352 | 5.124 | 40.857 |
| 2007 | 21,630 | 0.455 | 0.407 | 9,841.65 | 106,402.2 | 97,529 | 39,694.3 | 226,707.9 | 5.579 | 40.636 |
| 2008 | 22,810 | 0.407 | 0.363 | 9,283.7 | 115,685.9 | 91,672 | 33,276.9 | 242,408.9 | 5.986 | 40.496 |

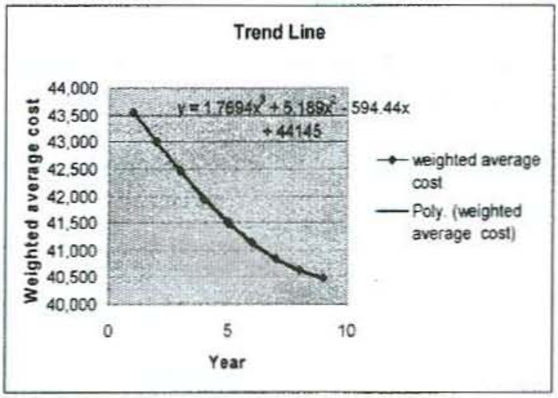


**Figure 1. Time vs. weighted average cost**

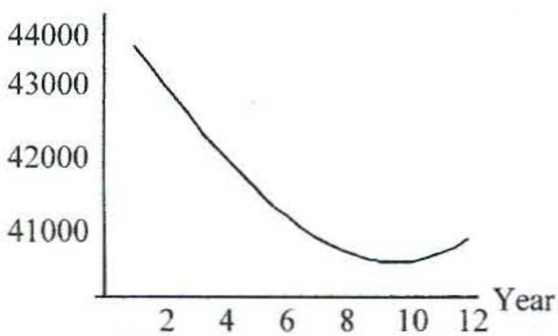


**Figure 2. Minimum weighted average cost**

## Discussion

If the operating and maintenance cost decrease or remain constant with time, the best policy is never to replace the item by a new one. However, this condition is hardly met with in practice. If these costs are fluctuating with time, the item should be replaced only when they are increasing. Generally all costs that depend upon the choice or age of the equipment must be taken in to account while analyzing the decision of its replacement. However, in special situations, certain costs may not be considered. For example, costs such as labour costs, electricity costs that do not change with the age of the equipment may not be included in the calculations. In actual practice; this type of replacement problem may be further complicated by prevailing tax laws.

## References

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