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# IMPACT OF WHITE NOISE ON THE CONVERGENCE OF THE TIME PATH OF THE PRICE FUNCTION IN DYNAMIC MARKET MODELS

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An economic model is a theoretical framework, and mathematics is inherent to it via equations and inequalities. In this study, the key ingredients of the model are $Q_d(t)$, $Q_s(t)$ in the usual notation, and the white noise $\xi(t)$ as a generalized function, that are specified as exogenous variables. The other three endogenous variables are price function $P(t)$, and the white noise multipliers $c(t)$ and $\gamma(t)$. The usual assumption is that the equilibrium is attained in the market if and only if the excess demand $E_d = (Q_d - Q_S)$ is zero; i.e., if and only if the market is cleared. The objective of this study is to investigate the convergence of the time path of the price function $P(t)$ and the stability of the intertemporal equilibrium price in the presence of white noise, when $E_d \neq 0$.

The model will be formulated under the assumptions that, the demand function $Q_d$ is a linear decreasing function in $P(t)$ and the supply function $Q_S$ is a linear increasing function in $P(t)$; white noise $\xi(t)$ is directly proportional to $\frac{dW}{dt}$, where $W(t)$ is the Wiener-Process adapting the white noise is to the system via $Q_d$ and $Q_S$ by linear superposition. In addition to the above, the rate of price change at any moment is directly proportional to the excess demand $E_d$. Mathematising the above assumptions and after some algebraic manipulations the governing stochastic differential equation (SDE) of the market model takes the form

$\frac{dP}{dt} = j[(a+\alpha)-(b+\beta)P]+[c-\gamma]\frac{dW}{dt}$. Here $P(0) = P_0(>0)$ and $j, a, b, \alpha, \beta$ are positive constants; $c$ and $\gamma$ are time-dependent continuous parameters.

The solution of the SDE with $l = a+\alpha$ and $m = b+\beta$ takes the form,

$P(t) = \left\{\left[P_0 - \left(\frac{l}{m}\right)\right]e^{-jmt} + \left(\frac{l}{m}\right) + \int_0^t e^{-jmt(1-\frac{s}{t})} n(s)dW(s)\right\}$. Thus the expectation of the price function $E(P(t))$ converges to the level of intertemporal equilibrium price $\bar{P} = \left(\frac{a+\alpha}{b+\beta}\right)$, assuring that the equilibrium price of the market model is stochastically stable.