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ORTHOGONAL SPHERICAL HARMONICS IN CUBED SPHERICAL COORDINATES AND APPLICATIONS TO POISSON EQUATION IN 2-SPHERE

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Computation of function values in a grid on the surface of the sphere and the integration of partial differential equations are important in many areas such as weather forecasting, global data analysis, and wave propagation. Most of these computations are performed under the spherical polar coordinate system for three dimensional space which results polarized grid mesh where the grid points converge at the north and south poles. Hence, computing discrete function values requires concentrated computations near the poles where the results are less important in practical situations. Besides, artificial singularities occurring due to these poles in solving partial differential equations using finite element method, for example, bring further difficulties to the computational scientists.

Avoiding polar singularities have gained some attention among many researches recently. Methods such as stereographic coordinate patches, rectangular coordinate system near the origin and polar coordinates away from the origin for two and three dimensional problems, artificial boundary with an artificial boundary condition around the polar origin in the domain of the Helmholtz equation have been practiced in previous works. Also, a new non-polar coordinate system for the three dimensional space composing six non-orthogonal coordinate systems for six faces on the unit sphere was defined and the construction of a fully orthogonal spherical harmonics for the system considering the surface Laplacian eigenvalue problem and the symmetric properties of the resulting differential equation was tried.

In this work, an orthogonal system of spherical harmonics in the non-polar coordinate system was constructed and their functional properties such as norms and orthonormality were analyzed. Then, by repartitioning the domain of the unit cube into four quadrants, a smoothly continuous eigenfunction on the 2-sphere was constructed and it was realized that these can be computed efficiently at grid points for discrete spherical harmonics transforms. Finally, these spherical harmonics were used to solve Poisson equations in the 2-sphere and some computations of finite Fourier series as immediate applications of the new orthogonal spherical harmonics were carried out.