CATASTROPHE VERSUS DUAL STABILIZATION POLICIES IN NATIONAL INCOME MODELS

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The study of dynamic stability of national income models is an interesting venture when the model is experiencing exogenous impulsive economic forces. These forces may generate catastrophes in the model causing deviations in the expected time path of the income function $Y(t)$ from its convergent point. The objective of this study is to investigate the sufficient conditions on the multipliers of the stabilization policies to prevent possible catastrophes within the model.

We assume that the national income Y is initially at the desired level and an exogenous decrease in aggregate demand D occurs. The basic dynamic mechanism is the multiplier where producers react to the excess demand by making adjustments in the output.

In this model, the three policies, Proportional, Derivative and Impulsive are incorporated into the model and the governing equation of the impulsive dual-stabilization model takes the form $Y'' + (\alpha l + \beta)Y' + \alpha\beta l Y - \alpha\beta G^* = -\alpha\beta$, where $0 < l < 1$ and $\alpha, \beta > 0$. The three policies make $G^* = -f_p Y - f_d Y' - f_i \int_{a-\varepsilon}^{a+\varepsilon} \delta(t-a) Y(t)\, dt$, where f_p and f_d are positive multipliers.

Using Laplace transform methods, we get the time-path of the income function

$$Y(t) = -\frac{\alpha\beta I}{s_1 s_2} - e^{s_1 t}\left[\frac{\alpha\beta I + s_1\alpha}{s_1(s_1 - s_2)}\right] + e^{s_2 t}\left[\frac{\alpha s_2 + \alpha\beta I}{s_2(s_1 - s_2)}\right]$$

where $I = 1 + f_i Y(a)$; and the values s_1 and s_2 are the zeros of the quadratic function $f(s) = s^2 + ((\alpha l + \beta + \alpha\beta f_d)s +)(\alpha\beta l + \alpha\beta f_p)$. Via the null-discriminant of $f(s)$ and using the Descartes' rule we can show that the national income model is free from catastrophes, and the income function $Y(t)$ is asymptotically stable provided the ratio $\frac{f_d}{f_p}$ of the multipliers of the dual policies exceeds the maximum value of $\frac{1}{\beta}$ and $\frac{1}{\alpha l}$.