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Lehmer's Conjecture and Minimal Absolute Height of the Algebraic Numbers of Cubic Polynomials with All Real Roots

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The Mahler measure of a polynomial with integer coefficients is defined to be the products of the absolute values of roots outside the unit circle and the absolute value of the leading coefficient of the polynomial. An algebraic number is a root of a non-zero polynomial in one variable with rational coefficients.

If an algebraic number α is a root of an irreducible integer polynomial of degree d, then the Mahler measure of α is defined as the Mahler measure of the polynomial. The absolute height of α, $H(\alpha)$, is defined $(M(\alpha))^{1/d}$ as, where $M(\alpha)$ is the Mahler measure of α.

In 1933, D. H. Lehmer raised his famous question referred to in literature as "Lehmer's conjecture" on the lower bound for $H(\alpha)$ which questions the existence of a constant $c > 1$, such that $H(\alpha)^d \geq c$, when α is not a root of unity. Lehmer has established that $c = 1.117628..$, where c is the largest real root of the polynomial $l(x) = 1 + x - x^3 - x^4 - x^5 - x^6 - x^7 + x^9 + x^{10}$. It is believed that c cannot be made arbitrarily close to 1.

The question posed on the existence of a lower bound for the absolute height of an algebraic number has attracted much attention after Lehmer's conjecture. Several important results have been established in the direction of the proof of his conjecture. In this endeavour, the absolute height of algebraic numbers corresponding to the set of irreducible integer polynomials of degree three having all real roots is investigated. In this effort, the greatest lower bound for the absolute height of the algebraic number associated with the class of polynomials under consideration was computed. Also, the irreducible polynomials associated with this value were found.