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FIRST ORDER OPTIMALITY CONDITIONS FOR MATHEMATICAL PROGRAMMING WITH *n*-SET FUNCTIONS

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The concept of optimizing set functions arises in various areas such as fluid flow, electrical insulator design, optimal plasma confinement, first and second order optimal conditions, and Fenchel duality theorem for set functions in Mathematics, Engineering, and Statistics. The optimization of set functions becomes difficult when a feasible domain which is neither open nor convex, but actually nowhere dense is considered. In 1979, Robert J. T. Morris overcame these difficulties by means of properly defined notions of convexity and differentiability of a set function. He considered the optimization problem in measure space (X, A, μ). The following example illustrates such a problem.

Rainfall in a region R is distributed according to the function r(x,y) where x is the longitude and y is the latitude of R. It is desired to plant a crop which has a per-acre yield of p(r) where r is the rainfall. The cost of planting per unit area is K and the area to be planted must not exceed A. The return realized from the total production p is u(p). The optimization problem is to choose a subregion for planting, that is, a set $\Omega \subset R$, such that the net return

$$u(\int_{\Omega} p(r(x,y))dxdy) - K\, measure(\Omega)$$

is maximized subject to the constraint:

$$measure(\Omega) \leq A.$$

In this report the convex set of A^n, the *n*-fold product of *A*, is introduced and the local convexity of an *n*-set function is defined. Differentiable convex *n*-set functions are considered and a necessary condition for an *n*-set function to be a differentiable convex *n*-set function and a sufficient condition for an *n*-set function to be a locally convex *n*-set function are derived. Finally, a first order optimality condition for mathematical programming with *n*-set functions is obtained.