CATASTROPHES AND STABILIZATION POLICIES IN ECONOMIC DYNAMICS

A PROJECT REPORT PRESENTED BY

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to the Board of Study in Mathematics of the

POSTGRADUATE INSTITUTE OF SCIENCE

in partial fulfillment of the requirement

for the award of the degree of

MASTER OF SCIENCE IN INDUSTRIAL MATHEMATICS

of the

UNIVERSITY OF PERADENIYA

SRILANKA

2007

ABSTRACT

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This research study is carried out predominantly to identify catastrophes and chaos, which can lead to complexities in the behaviour of economic functions in some dynamic models (micro/macro) related to the field of mathematical economics. As a micro model a detailed study of the foreign exchange market model has carried out when the supply function is a quadratic and the demand function is linear in the spot exchange rate $r(t)$. In this situation, it is noted that $r(t)$ is determined by the functional form $r = q\cot\underline{h}\left(Avqt + k\right) - p$, where p, q , A and v are exogenous parameters and k is an endogenous parameter. The graphical analysis of $r(t)$ shows that the asymptotical line is shifting along the time axis for different values of k and the model exhibits, out of seven, only fold catastrophe.

The influence of potential value of government demand G^* in national income models (NIM) can create abnormalities in the national income function $Y(t)$. In the presence of stabilization policies, it may be possible that national income (macro) models tend to behave well in the long run without generating catastrophes in the model. Thus it is an interesting task to search for necessary and sufficient conditions to prevent catastrophes by incorporating stabilization polices to national income models. In this direction, five types of models are investigated,

applying proportional stabilization policy, derivative stabilization policy and integral stabilization policy in the following manner:

(i) Proportional Stabilization Policy Model (PSP Model); Here the coefficient of proportional proportionality is denoted by f_p.

(ii) Mixed Proportional Derivative Stabilization Policy Model (MPDSP Model); Here the coefficient of derivative proportionality is denoted by f_d.

(iii) Integral Stabilization Policy Model (ISP Model); Here the coefficient of integral proportionality is denoted by f_i.

(iv) Impulsive Integral Stabilization Policy Model (IISP Model) and

(v) Triple Stabilization Policy Model (TSP Model); which is an extension of the IISP Model.

All these stabilization models are governed by the differential equation $\frac{d^2Y}{dt^2} + (\alpha l + \beta)\frac{dY}{dt} + \alpha\beta lY - \alpha\beta G^* = -\alpha\beta$, in the usual notation for the national income policy models. In PSP and MPDSP models, national income function $Y(t)$ converges to $-\frac{1}{1+f_p}$. In the ISP, $Y(t)$ exhibits explosive behaviour leading to chaos in the long term. IISP is quiet interesting as an extension of ISP model and it is relatively close to economic scenarios, which frequently appear in third world nations. This model exhibits catastrophes provided the marginal propensity to spend of the private sector reaches $\left(1-\frac{\beta}{\alpha}\right)$, where α and β are reaction coefficients representing adjustments factors of the national income policy model. In the triple stabilization policy model which is a supper - position of (i), (ii) and (iv), catastrophe occurs when the ratio of the coefficient of proportionalities $\frac{f_p}{f_d}$ **exceeds** the maximum value of αl and β.