Modified Stepping Stone Algorithm for Solving Large-Scale Balanced Transportation Problems

B.M.R.K. Basnayake and P.C. Perera

Department of Engineering Mathematics, Faculty of Engineering
University of Peradeniya

Introduction

The optimal solution to a transportation problem is sought by employing the Stepping Stone algorithm(SS), starting from some initial basic solution. Some attempts have been made et al Kèri to reduce high computational time associated with solving of transportation problems involving very large number of sources and destinations. In this endeavor, it is aimed at developing an algorithm which outperforms all Stepping Stone like algorithms in terms of computational time required for solving large-scale balanced transportation problems. The algorithm that is devised and tested in this work, is called Modified Stepping Stone algorithm(MSS).

Derived Stepping Stone algorithm (DSS)

A balanced transportation problem with m and n sources and destinations can be formulated as Minimization of $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$ such that $\sum_{i=1}^{m} x_{ij} = s_j$ for $j = 1, 2, ..., n$ and $\sum_{j=1}^{n} x_{ij} = d_i$ for $i = 1, 2, ..., m$. In the case of DSS, at k^{th} iteration, for each open cell (i, j) for which $z_{ij} - c_{ij}$ is negative, instead of $z_{ij} - c_{ij}$, it is considered $M_{ij}^{(k)}(z_{ij} - c_{ij})$, where $M_{ij}^{(k)}$ is the maximum value that can be assigned for the cell (i, j) with the solution at the k^{th} iteration.

There exist examples for which DSS requires approximately 20% more steps than SS. See graph 1. Nevertheless, DSS demonstrates relatively faster convergence if for $\frac{1}{5} < r < \frac{1}{2}$,

$$\frac{card(\{(i,j) : z_{ij} - c_{ij} < 0\})}{(m-1)(n-1)} > r \text{ and } \frac{\sigma_s}{\bar{s}} \ll 1,$$

where $S = \{c_{ij} - z_{ij} : z_{ij} - c_{ij} < 0\}$, $\bar{s}$ and σ_s are respectively the mean and the standard deviation of the set values of S. It can be shown that $card(\{(i,j) : z_{ij} - c_{ij} < 0\})$ and $\frac{\sigma_s}{\bar{s}}$ are functions of $mn - m - n + 1$ and the iteration number k. It should be noted that the derivation of closed forms of $card(\{(i,j) : z_{ij} - c_{ij} < 0\})$ and $\frac{\sigma_s}{\bar{s}}$ may be extremely tedious or impossible. Despite the unavailability of closed forms of the above functions, it can be shown that

$$\frac{card(\{(i,j) : z_{ij} - c_{ij} < 0\})}{(m-1)(n-1)},$$

decreases and

$$\frac{\sigma_s}{\bar{s}},$$

increases as the number of iterations k becomes large. Since, m and n are fixed for a given problem, it follows that

$$card(\{(i,j) : z_{ij} - c_{ij} < 0\})$$

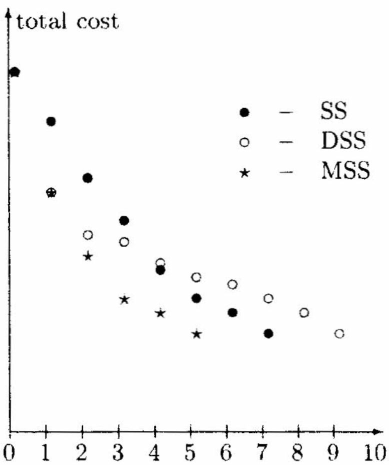
decreases with k.

Modified Stepping Stone algorithm (MSS)

The above facts motivates us to revert to the SS after the first few iterations by DSS, shows the most favorable results in terms of speed of convergence. According to the aforementioned description, tt turns out that the combined application of SS and DSS which is, in fact, MSS, produces the most favorable results. Topologically, the basic difference between the SS and MSS is that while the former always searches for the direction of the steepest decent, the latter choses the direction along which the highest fall is possible on the simplex $\hat{S} \subset \mathbb{N}^{mn}$.

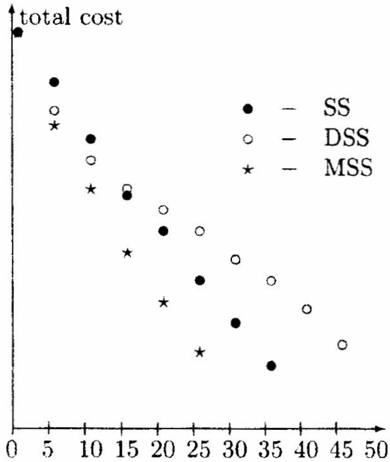
Results

For the sake of brevity, the results obtained from SS. DSS and MSS for a balanced transportation problem with 10 sources and 8 destinations are given in Graph 1.



Graph 1

For large scale balanced transportation problems the behaviors of SS and MSS turn out to be significantly different from each other. Consider the results obtained for a balanced transportation problem with 200 sources and 150 destinations which are provided in Graph 2.



Graph 2

Discussion

Let SS requires p iterations to converge to the optimal solution starting from an initial basic solution for a transportation problem with m and n sources and destinations respectively. Then, by applying DSS for the first $p\left\lfloor \frac{mn - m - n + 1}{\gamma} \right\rfloor$ followed by SS requires only αp iterations where $\gamma \propto (mn)^{\beta}$ for some $\beta > 0$ and $\alpha \in (0, 1 - f(m,n))$ with $0 < f(m,n)$ monotonically increasing with mn having unity as its least upper bound.

Conclusions

It can be concluded that MSS converges faster than SS for the large-scale balanced transportation problems. In terms of computational simplicity, MSS supersedes the algorithm developed by (Kèri et al 1972). It can also be concluded that the significance of this algorithm is enhanced when $m, n \rightarrow \infty$. Nevertheless, the following avenues are open for future work. Even though

$$card\,(\{(i,j) : z_{ij} - c_{ij} < 0\})$$

and $\frac{\sigma_s}{\overline{s}}$ are known to be the functions of $mn - m - n + 1$ and the iteration number k, models for those functions are yet to be determined. A statistical model may be viable though the formulation of precise expressions for the above functions is extremely tedious or impossible. More importantly, the parameters α, β, γ as well as the function $f(m, n)$ are to be determined.

References

Kèri, (1972) A modified stepping-stone algorithm for the Transportation Problem, *Int. J. of Theoretical and Applied Statistics*, 5 (2) 327-331