Proceedings of the Peradeniya University Research Sessions, Sri Lanka, Vol. 10, November 10, 2005

ORTHOGONAL SPHERICAL HARMONICS IN NON POLAR COORDINATES

H.M. NASIR AND N.A.R. JAYANATH

Department of Mathematics, Faculty of Science, University of Peradeniya

Spherical Harmonics plays an essential role in many applications, such as those in computational physics, chemistry, astronomy, biology, weather forecasting etc. In these areas much of the computational effort is directed towards the numerical solutions of partial differential equations in spherical geometry. The choice of spherical polar coordinates for the three dimensional space results in a polarized coordinate system.

Though widely used throughout the history of science, spherical polar coordinates display some polar singularities in computational considerations. This is due to the fact that the sample points on the sphere with respect to this coordinate system converge at the north and south poles. Hence, computing discrete values of functions defined on the sphere required concentrated computations near these poles. These computations are generally unnecessary because, in many modeling problems, information near poles is less important than in other areas. For example, weather prediction at the north and south poles on the earth is of less interest than in other parts of the globe. Besides, artificial singularities occurring at the north and south poles, bring further difficulties to the computational scientists when using numerical approximations..

In 2004, Nasir defined a new non polar coordinate system for the three dimensional space and displayed some simple computational results obtained for the spherical harmonics in 2-sphere. Later, in his undergraduate project (2005), Jayanath constructed a sequence of spherical harmonics in a non polar coordinate system and analyzed their basic algebraic properties. However, the spherical harmonics constructed were not fully orthogonal and hence need to be revised for efficient computations.

In the present work, we construct, by using recently developed (2001) elegant theory of differential equations having orthogonal polynomial systems, a fully orthogonal spherical harmonics for our problem.

The new orthogonal spherical harmonics are simple and can be efficiently computed for spectral computations using their Sheffer polynomial properties and hence fast algorithms for discrete spherical harmonics transforms can be devised on 2-sphere.